Lesson 003
Measures of Location

Friday, September 15

Survey Feedback

- **Note:** Questions mentioning "standard deviation", "variation", or "boxplots" will be covered next lesson.
- **Note:** Problem 32 had a missing histogram in the problem set.

- Comparing means and medians.
- How do we calculate the mean of categorical data?
- Questions 29, 31, and 32 in the problem set

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- **Sample mode** is the most common (set of) observation(s).

Example

Calculate the Mean, Median, and Mode of the Following Data

28	93	31	1
14	7	67	36
21	41	1	30

What is the mean of: 5, 5, 10, 20?

10	0%
7.5	0%
5	0%
40	0%

What is the median of: 5, 5, 10, 20?

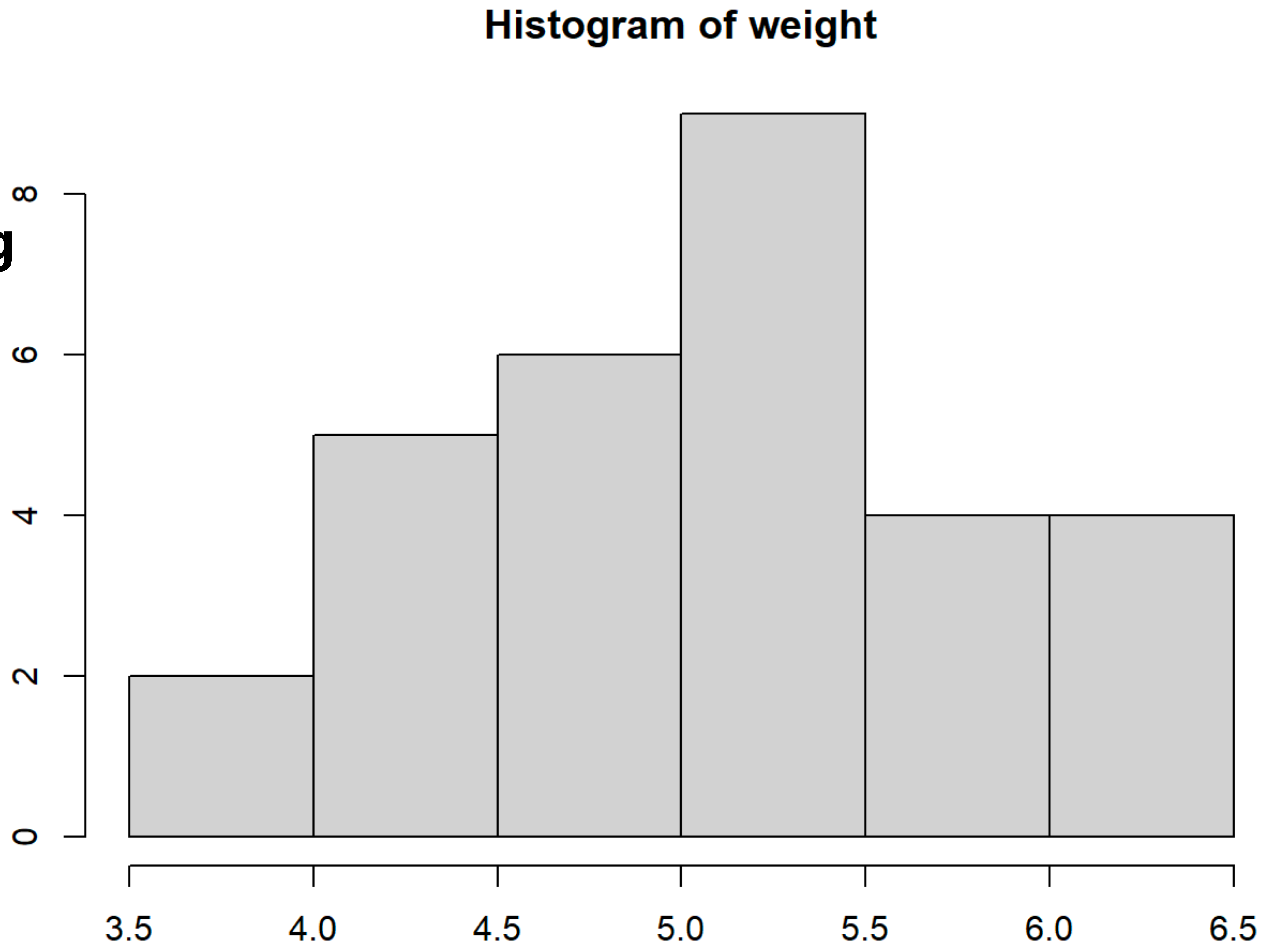
10	0%
7.5	0%
5	0%
40	0%

One batch from a manufacturing process had 15 items, with a mean weight of 10kg. A second batch had 10 items, with a mean weight of 9kg. What is the total weight of the two batches?

150kg	0%
240kg	0%
90kg	0%
9.6kg	0%

Example

Find the (approximate) mean from the following histogram



Example

Find the (approximate) mean from the following histogram

3.75	2	**5.25**	9
4.25	5	**5.75**	4
4.75	6	**6.25**	4

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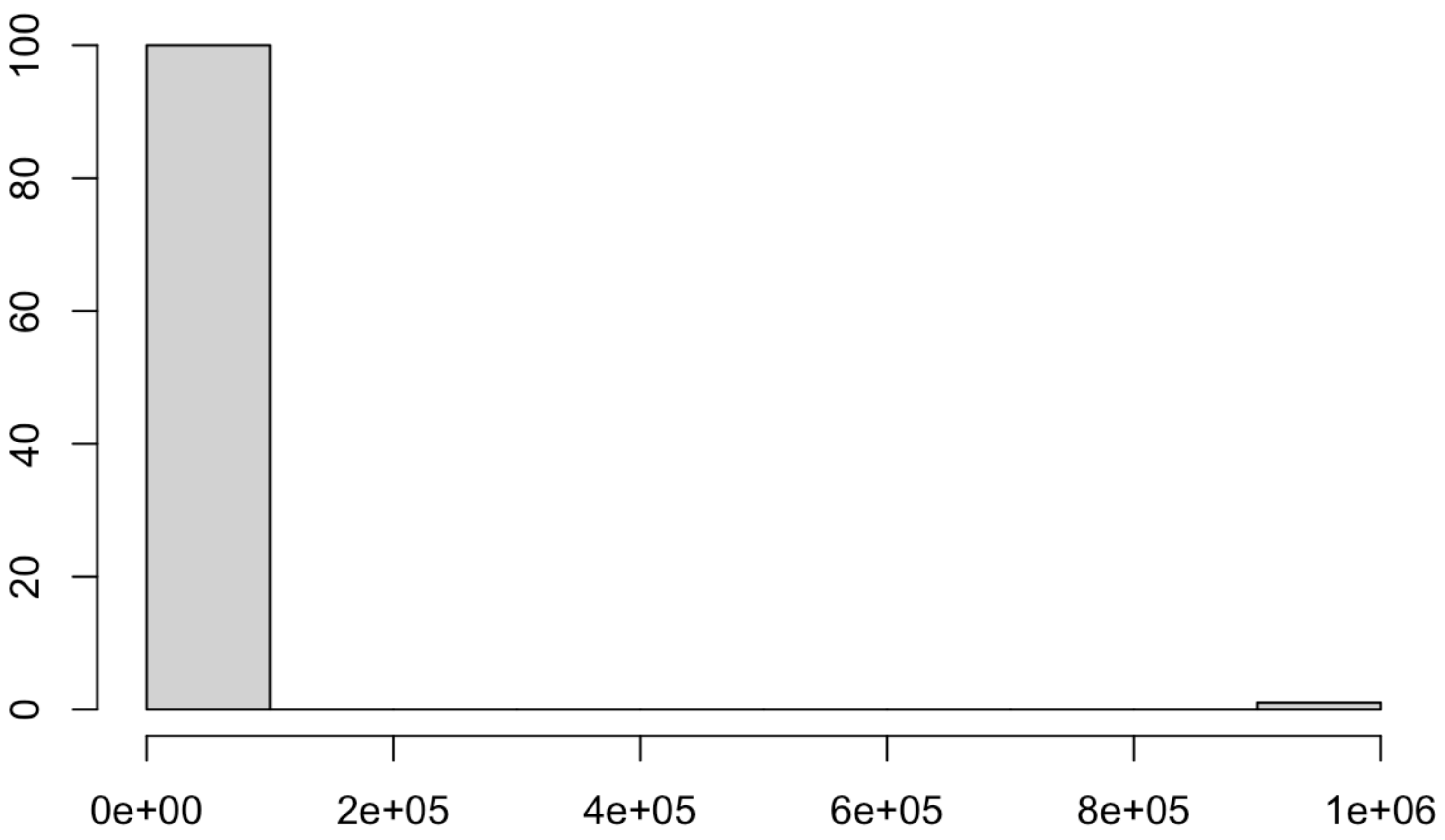
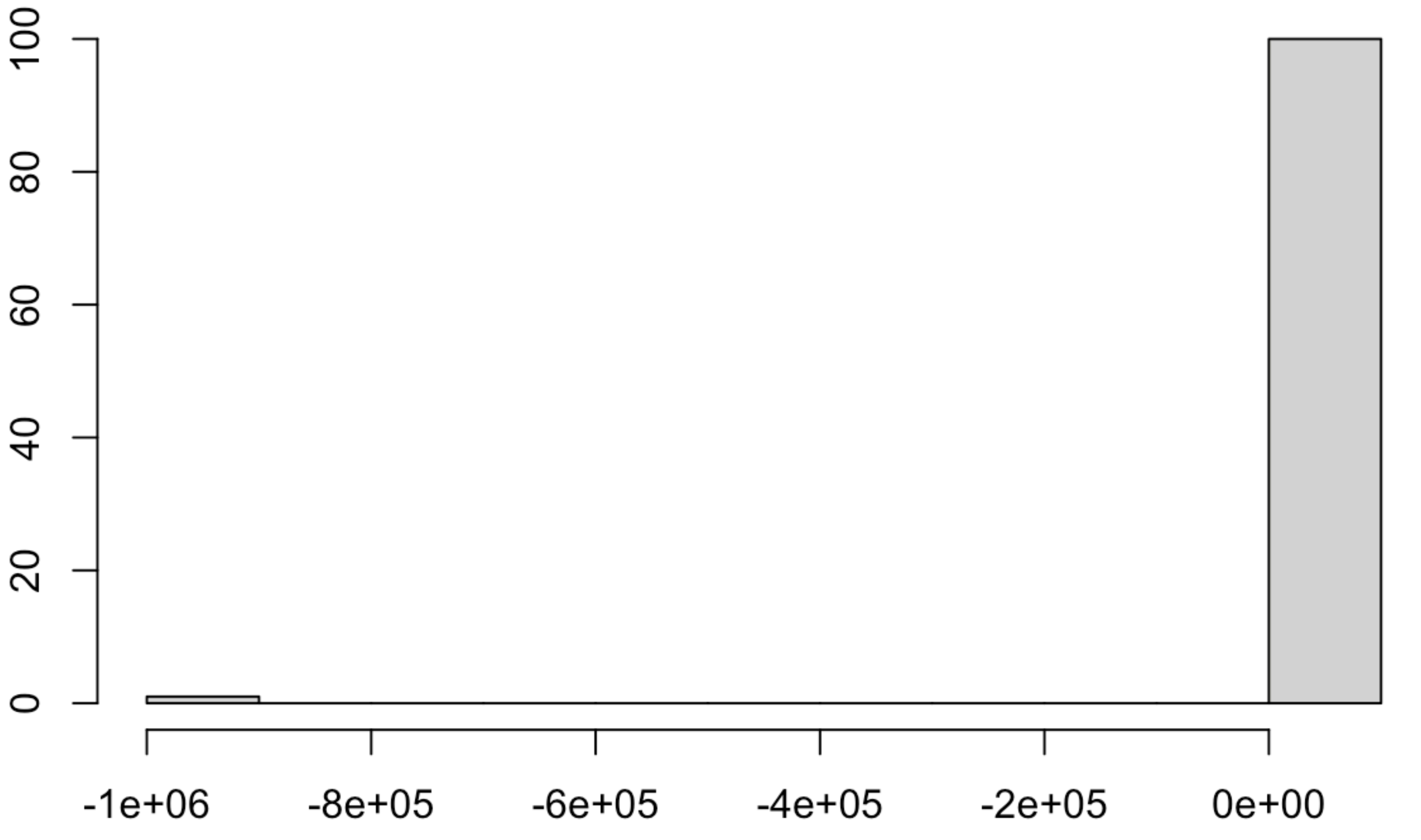
- If data are skewed, the mean is *pulled* towards the long tail of the distribution.
  - In this way, the mean is more sensitive to *skewed* outliers than the median.
- We generally prefer the median if data are skewed, and the mean otherwise.

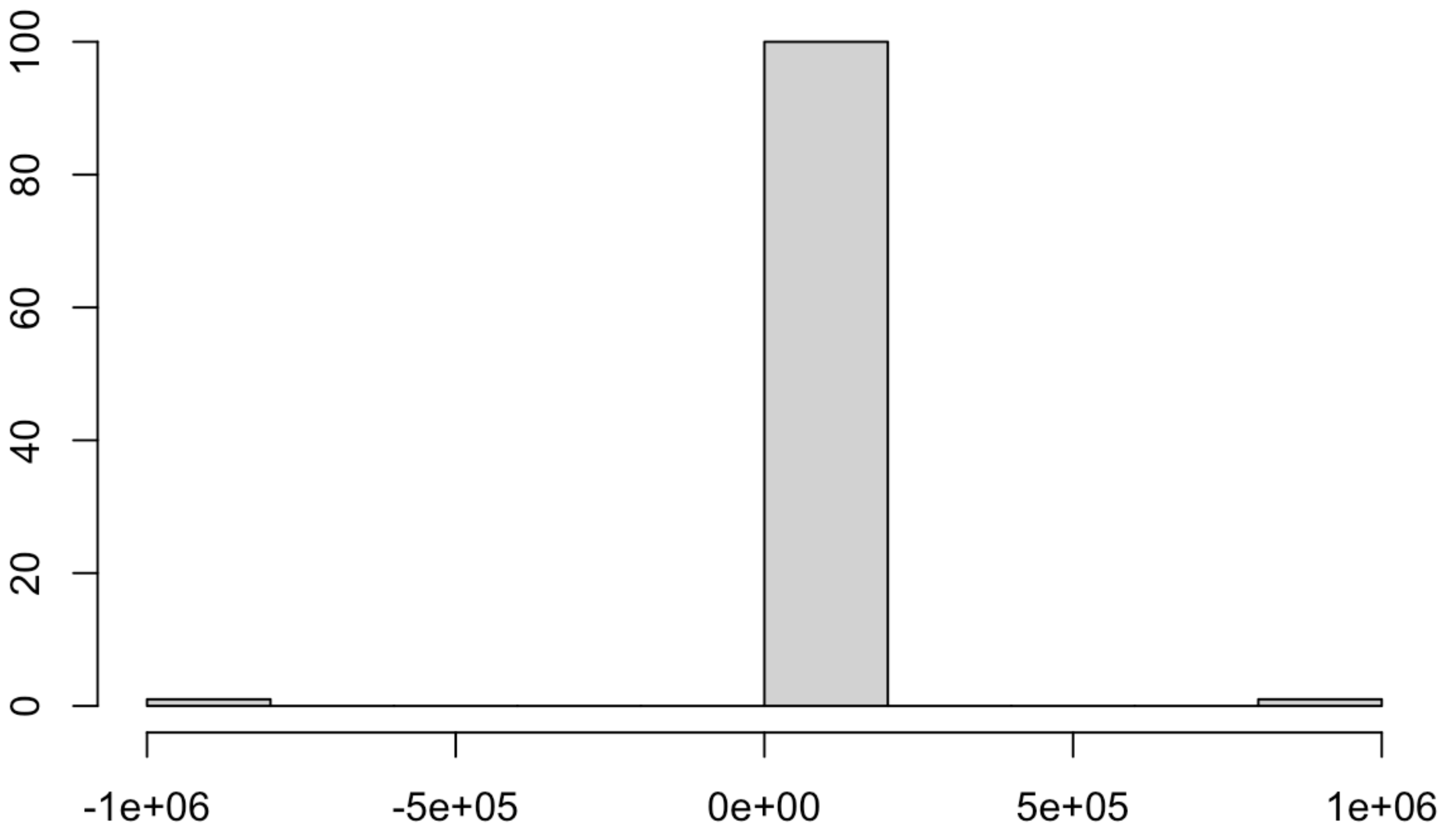
Example

Consider the mean and median of the following three sets of numbers (without calculation)

1, 1000000	1, -1000000	1, 1000000, -1000000







If you wanted a measure of location for household income, would you prefer the mean or median?

It does not matter since there will be no meaningful difference between them. 0%

The mean since the data are likely to be right skewed. 0%

The median since the data are likely to be right skewed. 0%

The median since the data are likely to have outliers. 0%

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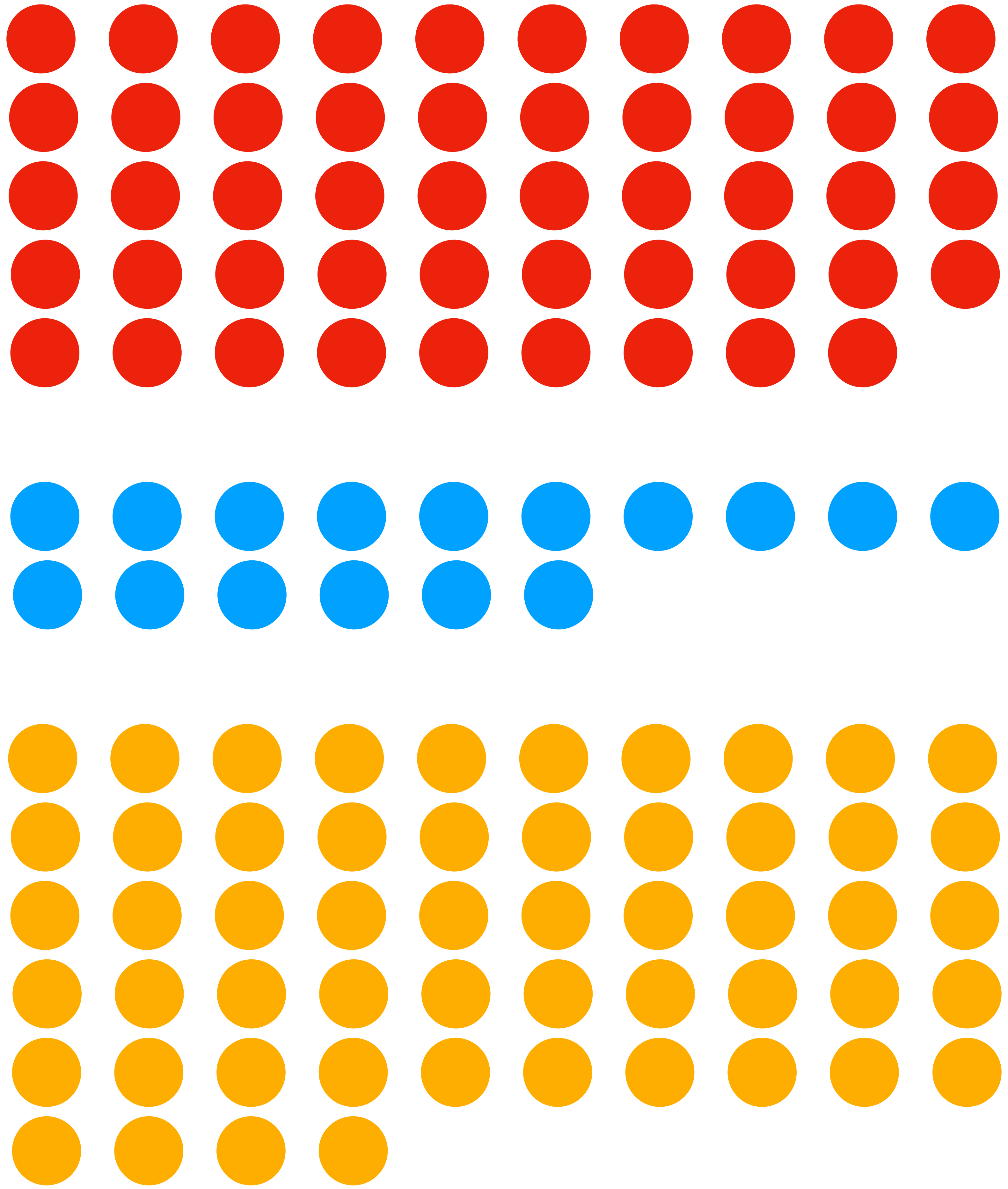
- Define $z_{i,j} = I(x_i = c_j)$, where $I(\cdot)$ is an indicator function.
- Then, we can write the j-th sample proportion as

$$p_j = \bar{z}_{\cdot,j} = \frac{1}{n} \sum_{i=1}^{n} z_{i,j}.$$

Example

Compute the Sample Proportion of 'Red'

	Count
Red	49
Blue	16
Yellow	54
	119



1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1

0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
0 0 0 0

$$49 / 119 = 0.41176$$

In the data from Spotify, 550 of the 953 songs were in a major key, the rest were in a minor key. What proportion of songs were in a minor key?

550/953 = 0.5771 — 0%

403/550 = 0.7327 — 0%

403/953 = 0.4229 — 0%

550/403 = 1.3648 — 0%